

Finite field-dependent BRST symmetry for ABJM theory in $\mathcal{N} = 1$ superspace

Mir Faizal¹

*Department of Physics and Astronomy, University of Waterloo, Waterloo,
Ontario N2L 3G1, Canada*

Sudhaker Upadhyay² and Bhabani Prasad Mandal³

*Department of Physics, Banaras Hindu University,
Varanasi-221005, India*

Abstract

In this paper we analyse the ABJM theory in $\mathcal{N} = 1$ superspace. Firstly we study the linear and non-linear BRST transformations for the ABJM theory. Then we derive the finite field dependent version of these BRST (FFBRST) transformations. Further we show that such FFBRST transformations relate the generating functional in linear gauge to the generating functional in the non-linear gauge of ABJM theory.

1 Introduction

According to the AdS/CFT correspondence this superconformal field theory is dual to the eleven dimensional supergravity on $AdS_4 \times S_7$. Apart from a constant closed 7-form on S^7 , $AdS_4 \times S_7 \sim SO(2,3) \times SO(1,2)/SO(8) \times SO(7) \subset OSp(8|4)/SO(1,3) \times SO(7)$. So, the dual superconformal field theory to the eleven dimensional supergravity on $AdS_4 \times S_7$ has $OSp(8|4)$ realized as $\mathcal{N} = 8$ supersymmetry. This theory also has eight gauge valued scalar fields, sixteen physical fermions and the gauge fields of this theory do not have any on-shell degrees of freedom. All these properties are satisfied by a theory called the BLG theory [1, 2, 3, 4, 5]. The BLG theory is based on gauge symmetry generated by a Lie 3-algebra rather than a Lie algebra. So, far the only known example of a Lie 3-algebra is $SO(4) \sim SU(2) \times SU(2)$, and it corresponds to two M2-branes. It has not been possible to increase the rank of the gauge group.

It has been possible to construct a superconformal gauge theory called the ABJM theory [6, 7, 8, 9]. The ABJM theory only has $\mathcal{N} = 6$ supersymmetry. However, it considers with the BLG theory for the only known example of the Lie 3-algebra and so its supersymmetry is expected to get enhanced to full $\mathcal{N} = 8$ supersymmetry [10]. The gauge sector is described by two Chern-Simons theories with levels k and $-k$. The matter fields in the ABJM theory are in the bi-fundamental representation of the gauge group $U(N)_k \times U(N)_{-k}$ and the gauge fields are in the adjoint representation. The ABJM theory has been studied in $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superspace formalism [11, 12, 13]. The ABJM theory has also been studied in harmonic superspace [14, 15]. However, in this paper, we will analyse the ABJM theory in $\mathcal{N} = 1$ superspace formalism. The BRST and the anti-BRST symmetries for the ABJM theory have been studied in both linear and non-linear gauges [16].

The infinitesimal BRST transformations have been generalized to finite field dependent BRST (FFBRST) originally in [17] and further generalized to construct finite field dependent anti-BRST (FFanti-BRST) transformations in [18]. Similar generalizations have also been made recently in [19, 20]. This is done by first making the infinitesimal global parameter occurring in the BRST or the anti-BRST transformations depend on fields occurring in the theory. Then this field dependent parameter is integrated to obtain the FFBRST and anti-FFBRST transformations. Even though, these finite transformations are

¹ f2mir@uwaterloo.ca

² sudhakerupadhyay@gmail.com

³ bhabani.mandal@gmail.com

a symmetry of the quantum action, they are not a symmetry of the functional measure. They can thus be used to relate a theory in one gauge to the same theory in a different gauge [19]-[28]. So, FFBRST transformations can be used to overcome a problem that a theory suffers from in a particular gauge. This can be done by first calculating the required quantity in a gauge in which that problem does not exist, and then using the FFBRST transformation to transform it to the required gauge. Thus, in Yang-Mills theory, FFBRST transformations have been used for obtaining the propagator in Coulomb gauge from the generating function in the Lorentz gauge [22]. The gauge-fixing and ghost terms corresponding to Landau and maximal Abelian gauge for the Cho-Faddeev-Niemi decomposed $SU(2)$ theory have also been generated using FFBRST transformation [29]. However, the linear and non-linear gauges of perturbative quantum gravity are connected at both classical and quantum level through FFBRST formulation [30]. The quantum gauge freedom described by gaugeon formalism has also been studied for quantum gravity [31] as well as for Higgs model [32] utilizing FFBRST technique. The FFBRST transformations are also studied in the context of lattice gauge theory [33] and relativistic point particle model [34].

The FFBRST transformation is used to relate the Gribov-Zwanziger theory to Yang-Mills theory in Landau gauge [35]. The problem of formulating the Gribov-Zwanziger theory beyond the Landau gauge is very delicate matter and substantial progress has been made recently towards the study of this problem [36, 37]. Thus, FFBRST transformations may give us an idea about the non-perturbative effects in a theory. This is very important from the M-theory point of view. This is because we may be able to understand the physics of multiple M5-branes by analysing non-perturbation effects in the ABJM theory [38]-[41]. The FFBRST transformations for the BLG theory has already been studied [42]. However, this limits the analysis to two M2-branes. If we want to analyse similar effects for multiple M2-branes, we need to analyse a similar system for ABJM theory. It may be noted that in analysing the FFBRST symmetry for the ABJM theory, we will need to introduce two finite field dependent parameters, which correspond to the gauge symmetries generated by $U(N)_k \times U(N)_{-k}$. As the matter fields transform in bi-fundamental representation of this gauge group, the matter sector mixes these two finite field dependent parameters. Thus, we need to generalize the ordinary FFBRST symmetry, to apply it on the ABJM theory. This is what we aim to do in this paper.

The paper is organized as follows. In Sec. 2, we discuss the preliminaries about ABJM theory in $\mathcal{N} = 1$ superspace. The BRST symmetry for various gauges are presented in Sec. 3. The FFBRST transformation for ABJM theory is developed in Sec. 4. In Sec. 5, we relate two arbitrary gauges of ABJM theory using FFBRST transformation.

2 ABJM Theory in $\mathcal{N} = 1$ Superspace

In this section we analyse ABJM theory on $\mathcal{N} = 1$ superspace. For this purpose, we begin with the Chern-Simons Lagrangian densities \mathcal{L}_{CS} , $\tilde{\mathcal{L}}_{CS}$ with gauge group's $U(N)_k$ and $U(N)_{-k}$ on $\mathcal{N} = 1$ superspace defined by

$$\begin{aligned}\mathcal{L}_{CS} &= \frac{k}{2\pi} \int d^2\theta \operatorname{Tr} \left[\Gamma^a \omega_a + \frac{i}{3} [\Gamma^a, \Gamma^b] D_b \Gamma_a + \frac{1}{3} [\Gamma^a, \Gamma^b] [\Gamma_a, \Gamma_b] \right], \\ \tilde{\mathcal{L}}_{CS} &= -\frac{k}{2\pi} \int d^2\theta \operatorname{Tr} \left[\tilde{\Gamma}^a \tilde{\omega}_a + \frac{i}{3} [\tilde{\Gamma}^a, \tilde{\Gamma}^b] D_b \tilde{\Gamma}_a + \frac{1}{3} [\tilde{\Gamma}^a, \tilde{\Gamma}^b] [\tilde{\Gamma}_a, \tilde{\Gamma}_b] \right],\end{aligned}\tag{1}$$

where ω_a and $\tilde{\omega}_a$ have following expression:

$$\begin{aligned}\omega_a &= \frac{1}{2} D^b D_a \Gamma_b - i [\Gamma^b, D_b \Gamma_a] - \frac{2}{3} [\Gamma^b, [\Gamma_b, \Gamma_a]], \\ \tilde{\omega}_a &= \frac{1}{2} D^b D_a \tilde{\Gamma}_b - i [\tilde{\Gamma}^b, D_b \tilde{\Gamma}_a] - \frac{2}{3} [\tilde{\Gamma}^b, [\tilde{\Gamma}_b, \tilde{\Gamma}_a]],\end{aligned}\tag{2}$$

with the super-derivative D_a defined by $D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b$.

In the component form the super-gauge connections Γ_a and $\tilde{\Gamma}_a$ are described by

$$\begin{aligned}\Gamma_a &= \chi_a + B\theta_a + \frac{1}{2}(\gamma^\mu)_a A_\mu + i\theta^2 \left[\lambda_a - \frac{1}{2}(\gamma^\mu \partial_\mu \chi)_a \right], \\ \tilde{\Gamma}_a &= \tilde{\chi}_a + \tilde{B}\theta_a + \frac{1}{2}(\gamma^\mu)_a \tilde{A}_\mu + i\theta^2 \left[\tilde{\lambda}_a - \frac{1}{2}(\gamma^\mu \partial_\mu \tilde{\chi})_a \right].\end{aligned}\quad (3)$$

The Lagrangian density of the matter fields is given by

$$\mathcal{L}_M = \frac{1}{4} \int d^2 \theta \text{Tr} \left[[\nabla_{(X)}^a X^{I\dagger} \nabla_{a(X)} X^I] + [\nabla_{(Y)}^a Y^{I\dagger} \nabla_{a(Y)} Y^I] + \frac{16\pi}{k} \mathcal{V} \right], \quad (4)$$

where

$$\begin{aligned}\nabla_{(X)a} X^I &= D_a X^I + i\Gamma_a X^I - iX^I \tilde{\Gamma}_a, \\ \nabla_{(X)a} X^{I\dagger} &= D_a X^{I\dagger} + i\tilde{\Gamma}_a X^{I\dagger} - iX^{I\dagger} \Gamma_a, \\ \nabla_{(Y)a} Y^I &= D_a Y^I + i\tilde{\Gamma}_a Y^I - iY^I \Gamma_a, \\ \nabla_{(Y)a} Y^{I\dagger} &= D_a Y^{I\dagger} + i\Gamma_a Y^{I\dagger} - iY^{I\dagger} \tilde{\Gamma}_a.\end{aligned}\quad (5)$$

Now, the gauge invariant Lagrangian density for ABJM theory with the gauge group $U(N)_k \times U(N)_{-k}$ on $\mathcal{N} = 1$ superspace is given by,

$$\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS} - \tilde{\mathcal{L}}_{CS}. \quad (6)$$

The gauge transformations are given by

$$\begin{aligned}\delta \Gamma_a &= \nabla_a \xi, & \delta \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{\xi}, \\ \delta X^I &= i\xi X^I - iX^I \tilde{\xi}, & \delta X^{I\dagger} &= i\tilde{\xi} X^{I\dagger} - iX^{I\dagger} \xi, \\ \delta Y^I &= i\tilde{\xi} Y^I - iY^I \xi, & \delta Y^{I\dagger} &= i\xi Y^{I\dagger} - iY^{I\dagger} \tilde{\xi},\end{aligned}\quad (7)$$

with the local parameters ξ and $\tilde{\xi}$. Here, the super-covariant derivatives ∇_a and $\tilde{\nabla}_a$ are defined by

$$\nabla_a = D_a - i\Gamma_a, \quad \tilde{\nabla}_a = D_a - i\tilde{\Gamma}_a. \quad (8)$$

Not all the degrees of freedom of this theory are physical as it is invariant under gauge transformations.

3 BRST Symmetry

In this section we will review the BRST symmetry for the ABJM theory in the $\mathcal{N} = 1$ superspace. Being gauge invariant, ABJM theory cannot be quantized without getting rid of these unphysical degrees of freedom. This is done by fixing the following gauge,

$$G_1 \equiv D^a \Gamma_a = 0, \quad \tilde{G}_1 \equiv D^a \tilde{\Gamma}_a = 0. \quad (9)$$

These gauge fixing conditions are incorporated at a quantum level by adding a gauge fixing term \mathcal{L}_{gf} and a ghost term \mathcal{L}_{gh} to the original classical Lagrangian. Here the gauge fixing term is given by

$$\mathcal{L}_{gf} = \int d^2 \theta \text{Tr} \left[ib(D^a \Gamma_a) + \frac{\alpha}{2} bb - i\tilde{b}(D^a \tilde{\Gamma}_a) - \frac{\alpha}{2} \tilde{b}\tilde{b} \right], \quad (10)$$

where b and \tilde{b} are the Nakanishi-Lautrup auxiliary fields. The Faddeev-Popov ghost term is given by

$$\mathcal{L}_{gh} = \int d^2 \theta \text{Tr} \left[i\bar{c} D^a \nabla_a c - i\tilde{c} D^a \tilde{\nabla}_a \tilde{c} \right]. \quad (11)$$

The sum of the original Lagrangian density with the gauge fixing and ghost terms is invariant under the following BRST transformations

$$\begin{aligned}
\delta_b \Gamma_a &= \nabla_a c \Lambda, & \delta_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c} \tilde{\Lambda}, \\
\delta_b c &= -[c, c] \Lambda, & \delta_b \tilde{c} &= -[\tilde{c}, \tilde{c}] \tilde{\Lambda}, \\
\delta_b \bar{c} &= b \Lambda, & \delta_b \tilde{\bar{c}} &= \tilde{b} \tilde{\Lambda}, \\
\delta_b b &= 0, & \delta_b \tilde{b} &= 0, \\
\delta_b X^I &= i\Lambda c X^I - iX^I \tilde{c} \tilde{\Lambda}, & \delta_b X^{I\dagger} &= i\tilde{\Lambda} \tilde{c} X^{I\dagger} - iX^{I\dagger} c \Lambda, \\
\delta_b Y^I &= i\tilde{\Lambda} \tilde{c} Y^I - iY^I c \Lambda, & \delta_b Y^{I\dagger} &= i\Lambda c Y^{I\dagger} - iY^{I\dagger} \tilde{c} \tilde{\Lambda},
\end{aligned} \tag{12}$$

where Λ and $\tilde{\Lambda}$ are the infinitesimal anticommuting parameters of transformation.

Now we analyse ABJM theory in non-linear gauge and therefore we define the Lagrangian density as follows

$$\begin{aligned}
\mathcal{L}_{NL} &= \mathcal{L}_c + \int d^2\theta \text{Tr} \left[\frac{\alpha}{2} b^2 + ibD^a \Gamma_a - iD^a \tilde{c} \nabla_a c - \frac{i}{2} D^a \Gamma_a [\tilde{c}, c] \right. \\
&+ \frac{\alpha}{8} [\tilde{c}, c]^2 - \frac{\alpha}{2} b [\tilde{c}, c] + iD^a \tilde{c} \nabla_a \tilde{c} - \frac{\alpha}{2} \tilde{b}^2 - i\tilde{b} D^a \tilde{\Gamma}_a \\
&\left. + \frac{i}{2} D^a \tilde{\Gamma}_a [\tilde{c}, \tilde{c}] - \frac{\alpha}{8} [\tilde{c}, \tilde{c}]^2 + \frac{\alpha}{2} \tilde{b} [\tilde{c}, \tilde{c}] \right].
\end{aligned} \tag{13}$$

We notice that the above Lagrangian density can be obtained by shifting the Nakanishi-Lautrup auxiliary fields as follows

$$b \rightarrow b - \frac{1}{2} [\tilde{c}, c], \quad \tilde{b} \rightarrow \tilde{b} - \frac{1}{2} [\tilde{c}, \tilde{c}]. \tag{14}$$

The BRST transformation, under which the effective action in non-linear gauge (13) is invariant, is given by

$$\begin{aligned}
\delta_b \Gamma_a &= \nabla_a c \Lambda, & \delta_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c} \tilde{\Lambda}, \\
\delta_b c &= -\frac{1}{2} [c, c] \Lambda, & \delta_b \tilde{c} &= -\frac{1}{2} [\tilde{c}, \tilde{c}] \tilde{\Lambda}, \\
\delta_b \bar{c} &= b \Lambda - \frac{1}{2} [\tilde{c}, c] \Lambda, & \delta_b \tilde{\bar{c}} &= \tilde{b} \tilde{\Lambda} - \frac{1}{2} [\tilde{c}, \tilde{c}] \tilde{\Lambda}, \\
\delta_b b &= -\frac{1}{2} [c, b] \Lambda - \frac{1}{8} [[c, c], \tilde{c}] \Lambda, & \delta_b \tilde{b} &= -\frac{1}{2} [\tilde{c}, \tilde{b}] \tilde{\Lambda} - \frac{1}{8} [[\tilde{c}, \tilde{c}], \tilde{c}] \tilde{\Lambda}, \\
\delta_b X^I &= i\Lambda c X^I - iX^I \tilde{c} \tilde{\Lambda}, & \delta_b X^{I\dagger} &= i\tilde{\Lambda} \tilde{c} X^{I\dagger} - iX^{I\dagger} c \Lambda, \\
\delta_b Y^I &= i\tilde{\Lambda} \tilde{c} Y^I - iY^I c \Lambda, & \delta_b Y^{I\dagger} &= i\Lambda c Y^{I\dagger} - iY^{I\dagger} \tilde{c} \tilde{\Lambda}.
\end{aligned} \tag{15}$$

Remarkably, the effective action is also found invariant under the another set of BRST symmetry (called as anti-BRST transformation) where roles of ghost and anti-ghost fields are interchanged. The anti-BRST transformation is written by

$$\begin{aligned}
\delta_{ab} \Gamma_a &= \nabla_a \bar{c} \bar{\Lambda}, & \delta_{ab} \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{\bar{c}} \tilde{\bar{\Lambda}}, \\
\delta_{ab} \bar{c} &= -\frac{1}{2} [\bar{c}, \bar{c}] \bar{\Lambda}, & \delta_{ab} \tilde{\bar{c}} &= -\frac{1}{2} [\tilde{\bar{c}}, \tilde{\bar{c}}] \tilde{\bar{\Lambda}}, \\
\delta_{ab} c &= -b \bar{\Lambda} - \frac{1}{2} [\bar{c}, c] \bar{\Lambda}, & \delta_{ab} \tilde{c} &= -\tilde{b} \tilde{\bar{\Lambda}} - \frac{1}{2} [\tilde{\bar{c}}, \tilde{c}] \tilde{\bar{\Lambda}}, \\
\delta_{ab} b &= -\frac{1}{2} [\bar{c}, b] \bar{\Lambda} + \frac{1}{8} [[\bar{c}, \bar{c}], c] \bar{\Lambda}, & \delta_{ab} \tilde{b} &= -\frac{1}{2} [\tilde{\bar{c}}, \tilde{b}] \tilde{\bar{\Lambda}} + \frac{1}{8} [[\tilde{\bar{c}}, \tilde{\bar{c}}], \tilde{c}] \tilde{\bar{\Lambda}}, \\
\delta_{ab} X^I &= i\bar{\Lambda} \bar{c} X^I - iX^I \tilde{\bar{c}} \tilde{\bar{\Lambda}}, & \delta_{ab} X^{I\dagger} &= i\tilde{\bar{\Lambda}} \tilde{\bar{c}} X^{I\dagger} - iX^{I\dagger} \bar{c} \bar{\Lambda}, \\
\delta_{ab} Y^I &= i\tilde{\bar{\Lambda}} \tilde{\bar{c}} Y^I - iY^I \bar{c} \bar{\Lambda}, & \delta_{ab} Y^{I\dagger} &= i\bar{\Lambda} \bar{c} Y^{I\dagger} - iY^{I\dagger} \tilde{\bar{c}} \tilde{\bar{\Lambda}}.
\end{aligned} \tag{16}$$

The above BRST and anti-BRST transformations satisfy the following algebra:

$$\delta_b^2 = 0, \quad \delta_{ab}^2 = 0, \quad \delta_b \delta_{ab} + \delta_{ab} \delta_b = 0. \quad (17)$$

With these BRST and anti-BRST transformations the Lagrangian density (13) can also be expressed as

$$\begin{aligned} \mathcal{L}_{NL} &= \mathcal{L}_c + \frac{i}{2} \delta_b \delta_{ab} \int d^2\theta \operatorname{Tr} \left[\Gamma_a \Gamma^a - \tilde{\Gamma}_a \tilde{\Gamma}^a - i\alpha \bar{c}c + i\alpha \tilde{c}\tilde{c} \right], \\ &= \mathcal{L}_c - \frac{i}{2} \delta_{ab} \delta_b \int d^2\theta \operatorname{Tr} \left[\Gamma_a \Gamma^a - \tilde{\Gamma}_a \tilde{\Gamma}^a - i\alpha \bar{c}c + i\alpha \tilde{c}\tilde{c} \right]. \end{aligned} \quad (18)$$

4 Finite Field Dependent Transformation

In this section we construct finite field dependent BRST transformation [17] of ABJM theory in $\mathcal{N} = 1$ superspace. To do that we first define two sets of generic fields as $\Phi_L^i(x, \kappa) \equiv (\Gamma_a, X^I, Y^I, c, \bar{c}, b)$ and $\Phi_R^i(x, \kappa) \equiv (\tilde{\Gamma}_a, \tilde{X}^I, \tilde{Y}^I, \tilde{c}, \tilde{\bar{c}}, b)$, here the parameter $\kappa : 0 \leq \kappa \leq 1$. Here $\Phi_L^i(x, 0), \Phi_R^i(x, 0)$ are the initial fields and $\Phi_L^i(x, 1), \Phi_R^i(x, 1)$ are the transformed fields.

The infinitesimal but field dependent BRST transformations can be written as [17]

$$\begin{aligned} \frac{d}{d\kappa} \Phi_L^i(x, \kappa) &= s \Phi_L^i(x) \epsilon_L[\Phi_L(x, \kappa)], \\ \frac{d}{d\kappa} \Phi_R^i(x, \kappa) &= s \Phi_R^i(x) \epsilon_R[\Phi_R(x, \kappa)]. \end{aligned} \quad (19)$$

where $\epsilon_L[\Phi_L(x)]$ and $\epsilon_R[\Phi_R(x)]$ are infinitesimal field dependent parameters. Now integrating the above equation from $\kappa = 0$ to $\kappa = 1$, we get the FFBRST transformation,

$$\begin{aligned} \Phi_L^i(x, 1) &= \Phi_L^i(x, 0) + s \Phi_L^i(x) \Theta_L[\Phi_L(x)], \\ \Phi_R^i(x, 1) &= \Phi_R^i(x, 0) + s \Phi_R^i(x) \Theta_R[\Phi_R(x)], \end{aligned} \quad (20)$$

where finite field dependent parameters are

$$\begin{aligned} \Theta_L[\Phi_L(x)] &= \int_0^1 d\kappa \epsilon_L[\Phi_L(x, \kappa)], \\ \Theta_R[\Phi_R(x)] &= \int_0^1 d\kappa \epsilon_R[\Phi_R(x, \kappa)]. \end{aligned} \quad (21)$$

Furthermore these finite parameters are calculated [17] as,

$$\begin{aligned} \Theta_L[\Phi_L(x)] &= \epsilon_L[\Phi_L(x)] \frac{\exp F_L[\Phi_L(x)] - 1}{F_L[\Phi_L(x)]}, \\ \Theta_R[\Phi_R(x)] &= \epsilon_R[\Phi_R(x)] \frac{\exp F_R[\Phi_R(x)] - 1}{F_R[\Phi_R(x)]}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} F_L &= \sum_i \frac{\delta \epsilon_L[\Phi_L(x)]}{\delta \Phi_L^i(x)} s \Phi_L^i(x), \\ F_R &= \sum_i \frac{\delta \epsilon_R[\Phi_R(x)]}{\delta \Phi_R^i(x)} s \Phi_R^i(x). \end{aligned} \quad (23)$$

Now the FFBRST transformations in the linear gauge are given by

$$\begin{aligned}
\delta_b \Gamma_a &= \nabla_a c \Theta_L, & \delta_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c} \Theta_R, \\
\delta_b c &= -[c, c] \Theta_L, & \delta_b \tilde{c} &= -[\tilde{c}, \tilde{c}] \Theta_R, \\
\delta_b \bar{c} &= b \Theta_L, & \delta_b \tilde{\bar{c}} &= \tilde{b} \Theta_R, \\
\delta_b b &= 0, & \delta_b \tilde{b} &= 0, \\
\delta_b X^I &= i\Theta_L c X^I - iX^I \tilde{c} \Theta_R, & \delta_b X^{I\dagger} &= i\Theta_R \tilde{c} X^{I\dagger} - iX^{I\dagger} c \Theta_L, \\
\delta_b Y^I &= i\Theta_R \tilde{c} Y^I - iY^I c \Theta_L, & \delta_b Y^{I\dagger} &= i\Theta_L c Y^{I\dagger} - iY^{I\dagger} \tilde{c} \Theta_R.
\end{aligned} \tag{24}$$

and the FFBRST transformations in the non-linear gauge are given by

$$\begin{aligned}
\delta_b \Gamma_a &= \nabla_a c \Theta_L, & \delta_b \tilde{\Gamma}_a &= \tilde{\nabla}_a \tilde{c} \Theta_R, \\
\delta_b c &= -\frac{1}{2}[c, c] \Theta_L, & \delta_b \tilde{c} &= -\frac{1}{2}[\tilde{c}, \tilde{c}] \Theta_R, \\
\delta_b \bar{c} &= b \Theta_L - \frac{1}{2}[\bar{c}, c] \Theta_L, & \delta_b \tilde{\bar{c}} &= \tilde{b} \Theta_R - \frac{1}{2}[\tilde{\bar{c}}, \tilde{c}] \Theta_R, \\
\delta_b b &= -\frac{1}{2}[c, b] \Theta_L - \frac{1}{8}[[c, c], \bar{c}] \Theta_L, & \delta_b \tilde{b} &= -\frac{1}{2}[\tilde{c}, \tilde{b}] \Theta_R - \frac{1}{8}[[\tilde{c}, \tilde{c}], \tilde{c}] \Theta_R, \\
\delta_b X^I &= i\Theta_L c X^I - iX^I \tilde{c} \Theta_R, & \delta_b X^{I\dagger} &= i\Theta_R \tilde{c} X^{I\dagger} - iX^{I\dagger} c \Theta_L, \\
\delta_b Y^I &= i\Theta_R \tilde{c} Y^I - iY^I c \Theta_L, & \delta_b Y^{I\dagger} &= i\Theta_L c Y^{I\dagger} - iY^{I\dagger} \tilde{c} \Theta_R.
\end{aligned} \tag{25}$$

The Jacobians for path integral measures in the expression of generating functionals are given by

$$\mathcal{D}\Phi^i_L = J_L[\Phi_L(\kappa)] \mathcal{D}\Phi^i_L(\kappa), \quad \mathcal{D}\Phi^i_R = J_R[\Phi_R(\kappa)] \mathcal{D}\Phi^i_R(\kappa). \tag{26}$$

So, the FFBRST transformations are not a symmetry of the generating functional. Now $J_L[\Phi_L(\kappa)]J_R[\Phi_R(\kappa)]$ can be replaced within the functional integral by $\exp(iS_{1L}[\Phi_L(\kappa)] + iS_{1R}[\Phi_R(\kappa)])$, if the following equations are satisfied,

$$\begin{aligned}
\int d^2\theta \operatorname{Tr} \left[\frac{1}{J_L(\kappa)} \frac{dJ_L(\kappa)}{d\kappa} - i \frac{dS_{1L}}{d\kappa} \right] &= 0, \\
\int d^2\theta \operatorname{Tr} \left[\frac{1}{J_R(\kappa)} \frac{dJ_R(\kappa)}{d\kappa} - i \frac{dS_{1R}}{d\kappa} \right] &= 0.
\end{aligned} \tag{27}$$

The infinitesimal changes in Jacobian's are given by

$$\begin{aligned}
\frac{1}{J_L(\kappa)} \frac{dJ_L(\kappa)}{d\kappa} &= - \int d^2\theta \operatorname{Tr} \mathcal{A}_L, \\
\frac{1}{J_R(\kappa)} \frac{dJ_R(\kappa)}{d\kappa} &= - \int d^2\theta \operatorname{Tr} \mathcal{A}_R,
\end{aligned} \tag{28}$$

where explicit expressions for \mathcal{A}_L and \mathcal{A}_R are given by

$$\begin{aligned}
\mathcal{A}_L &= \left[s\Gamma_a(x, \kappa) \frac{\delta\epsilon_L[\Phi_L(x, k)]}{\delta\Gamma_a(x, k)} - sc(x, k) \frac{\delta\epsilon_L[\Phi_L(x)]}{\delta c(x, k)} \right. \\
&\quad - s\bar{c}(x, k) \frac{\delta\epsilon_L[\Phi_L(x, k)]}{\delta\bar{c}(x, k)} + sb(x, k) \frac{\delta\epsilon_L[\Phi_L(x, k)]}{\delta b(x, k)} \\
&\quad \left. - sX^I(x, k) \frac{\delta\epsilon_L[\Phi_L(x, k)]}{\delta X^I(x, k)} + sY^I(x, k) \frac{\delta\epsilon_L[\Phi_L(x, k)]}{\delta Y^I(x, k)} \right], \\
\mathcal{A}_R &= \left[s\tilde{\Gamma}_a(x, \kappa) \frac{\delta\epsilon_R[\Phi_R(x, k)]}{\delta\tilde{\Gamma}_a(x, k)} - s\tilde{c}(x, k) \frac{\delta\epsilon_R[\Phi_R(x)]}{\delta\tilde{c}(x, k)} \right.
\end{aligned}$$

$$\begin{aligned}
& -s\tilde{c}(x, k) \frac{\delta\epsilon_R[\Phi_R(x, k)]}{\delta\tilde{c}(x, k)} + s\tilde{b}(x, k) \frac{\delta\epsilon_R[\Phi_R(x, k)]}{\delta\tilde{b}(x, k)} \\
& -sX^{I\dagger}(x, k) \frac{\delta\epsilon_R[\Phi_R(x, k)]}{\delta X^{I\dagger}(x, k)} + sY^{I\dagger}(x, k) \frac{\delta\epsilon_R[\Phi_R(x, k)]}{\delta Y^{I\dagger}(x, k)} \Big]. \tag{29}
\end{aligned}$$

Here we note that the conditions (27) provide us liberty to replace the Jacobians of path integral measure by the exponential of local functional within functional measure. Hence, the Jacobians amount a precise change in effective action of generating functional. One can also arrive at the same conclusion following the work in Ref. [19, 20].

5 Relating Different Gauges

We will now use FFBRST to relate the generating functional in the linear gauge to the generating functional in the non-linear gauge. If the gauge fixing condition in the linear gauge is denoted by $G_{1L}[\Gamma_a], G_{1R}[\tilde{\Gamma}_a]$ and the gauge fixing condition in the non-linear gauge is denoted by $G_{2L}[\Gamma_a], G_{1R}[\tilde{\Gamma}_a]$, then, the linear BRST transformations of $G_{1L}[\Gamma_a], G_{1R}[\tilde{\Gamma}_a]$ are denoted by sG_{1L}, sG_{1R} and the non-linear BRST transformations of $G_{2L}[\Gamma_a], G_{1R}[\tilde{\Gamma}_a]$ are denoted by sG_{2L}, sG_{2R} . We define the infinitesimal field dependent parameter as follows

$$\begin{aligned}
\epsilon_L[\Phi_L] &= i\gamma \int d^2\theta \text{Tr} [\bar{c}(G_{1L} - G_{2L})], \\
\epsilon_R[\Phi_R] &= -i\gamma \int d^2\theta \text{Tr} [\tilde{c}(G_{1R} - G_{2R})], \tag{30}
\end{aligned}$$

where γ is an arbitrary constant parameter.

Using definition given in (28), the change in Jacobian's can be calculated as follows,

$$\begin{aligned}
\frac{1}{J_L} \frac{dJ_L}{d\kappa} &= i\gamma \int d^2\theta \text{Tr} [bG_{1L} - bG_{2L} - (sG_{1L} - sG_{2L})\bar{c}], \\
&= i\gamma \int d^2\theta \text{Tr} [bG_{1L} - bG_{2L} + \bar{c}(sG_{1L} - sG_{2L})], \\
\frac{1}{J_R} \frac{dJ_R}{d\kappa} &= -i\gamma \int d^2\theta \text{Tr} [\tilde{b}G_{1R} - \tilde{b}G_{2R} - (sG_{1R} - sG_{2R})\tilde{c}], \\
&= -i\gamma \int d^2\theta \text{Tr} [\tilde{b}G_{1R} - \tilde{b}G_{2R} + \tilde{c}(sG_{1R} - sG_{2R})]. \tag{31}
\end{aligned}$$

Furthermore, the local functionals S_{1L} and S_{1R} involved in the Jacobians are defined as

$$\begin{aligned}
S_{1L} &= \int d^2\theta \text{Tr} [\xi_{1L}(\kappa)bG_{1L} + \xi_{2L}(\kappa)bG_{2L} \\
&\quad + \xi_{3L}(\kappa)\bar{c}sG_{1L} + \xi_{4L}(\kappa)\bar{c}sG_{2L}], \\
S_{1R} &= \int d^2\theta \text{Tr} [\xi_{1R}(\kappa)\tilde{b}G_{1R} + \xi_{2R}(\kappa)\tilde{b}G_{2R} \\
&\quad + \xi_{3R}(\kappa)\tilde{c}sG_{1R} + \xi_{4R}(\kappa)\tilde{c}sG_{2R}], \tag{32}
\end{aligned}$$

where $\xi_{iL}, \xi_{iR}, (i = 1, 2, 3, 4)$ are κ dependent arbitrary parameters which satisfy the following initial boundary conditions, $\xi_{iL}(\kappa = 0) = \xi_{iR}(\kappa = 0) = 0$. As all the fields depend on κ , so we can write

$$\begin{aligned}
\frac{dS_{1L}}{d\kappa} &= \int d^2\theta \text{Tr} [\xi'_{1L}bG_{1L} + \xi_{1L}bsG_{1L}\epsilon_L \\
&\quad + \xi_{2R}bsG_{2L}\epsilon_L + \xi'_{3L}\bar{c}sG_{1L} - \xi_{3L}bsG_{1L}\epsilon_L \\
&\quad + \xi'_{4L}\bar{c}sG_{2L} - \xi_{4L}bsG_{2L}\epsilon_L]
\end{aligned}$$

$$\begin{aligned}
& +\xi'_{4L}\bar{c}sG_{2L} - \xi_{4L}bsG_{2L}\epsilon_L + \xi'_{2L}bG_{2L}], \\
= & \int d^2\theta \text{Tr} [\xi'_{1L}bG_{1L} + \xi'_{2L}bG_{2L} \\
& +\xi'_{3L}\bar{c}sG_{1L} + \xi'_{4L}\bar{c}sG_{2L} \\
& +(\xi_{1L} - \xi_{3L})bsG_{1L}\epsilon_L + (\xi_{2L} - \xi_{4L})bsG_{2L}\epsilon_L], \\
\frac{dS_{1R}}{d\kappa} = & \int d^2\theta \text{Tr} [\xi'_{1R}\tilde{b}G_{1R} + \xi_{1R}\tilde{b}sG_{1R}\epsilon_R \\
& +\xi_{2R}\tilde{b}sG_{2R}\epsilon_R + \xi'_{3R}\tilde{c}sG_{1R} - \xi_{3R}\tilde{b}sG_{1R}\epsilon_R \\
& +\xi'_{4R}\tilde{c}sG_{2R} - \xi_{4R}\tilde{b}sG_{2R}\epsilon_R + \xi'_{2R}\tilde{b}G_{2R}], \\
= & \int d^2\theta \text{Tr} [\xi'_{1R}\tilde{b}G_{1R} + \xi'_{2R}\tilde{b}G_{2R} \\
& +\xi'_{3R}\tilde{c}sG_{1R} + \xi'_{4R}\tilde{c}sG_{2R} \\
& +(\xi_{1R} - \xi_{3R})\tilde{b}sG_{1R}\epsilon_R + (\xi_{2R} - \xi_{4R})\tilde{b}sG_{2R}\epsilon_R]. \tag{33}
\end{aligned}$$

The Jacobians of path integral measure can be written as $\exp(iS_{1L} + iS_{1R})$, when the following equations are satisfied,

$$\begin{aligned}
& \int d^2\theta \text{Tr} [(\xi'_{1L} - \gamma)bG_{1L} + (\xi'_{2L} + \gamma)bG_{2L} \\
& +(\xi'_{3L} - \gamma)\bar{c}sG_{1L} + (\xi'_{4L} + \gamma)\bar{c}sG_{2L} \\
& +(\xi_{1L} - \xi_{3L})bsG_{1L}\epsilon_L + (\xi_{2L} - \xi_{4L})bsG_{2L}\epsilon_L] = 0, \\
& \int d^2\theta \text{Tr} [(\xi'_{1R} + \gamma)\tilde{b}G_{1R} + (\xi'_{2R} - \gamma)\tilde{b}G_{2R} \\
& +(\xi'_{3R} + \gamma)\tilde{c}sG_{1R} + (\xi'_{4R} - \gamma)\tilde{c}sG_{2R} \\
& +(\xi_{1R} - \xi_{3R})\tilde{b}sG_{1R}\epsilon_R + (\xi_{2R} - \xi_{4R})\tilde{b}sG_{2R}\epsilon_R] = 0. \tag{34}
\end{aligned}$$

Equating the coefficients of the above expressions, and setting $\gamma = 1$, we get $\xi_{1L} = -\xi_{1R} = \kappa$, $\xi_{2L} = -\xi_{2R} = -\kappa$, $\xi_{3L} = -\xi_{3R} = \kappa$, $\xi_{4L} = -\xi_{4R} = -\kappa$. Now, if we add $S_1 = S_{1L}(\kappa = 1) + S_{1R}(\kappa = 1)$ to the original action in the non-linear gauge, we obtain the action in the linear gauge within a functional integral. So, under the FFBRST transformations the generating functional in the non-linear gauge transforms to the generating functional in the linear gauge. Similar computations can also be made following the work in Ref. [19, 20] to show that the FFBRST transformation amounts finite change in gauge-fixing fermion of the path integral.

6 Conclusion

In this paper we analysed the FFBRST transformations for the ABJM theory in $\mathcal{N} = 1$ superspace. We first have discussed the BRST for the ABJM theory in $\mathcal{N} = 1$ superspace. Then we have integrated the infinitesimal parameter in the BRST transformations to obtain the FFBRST transformations. As the ABJM theory contains two Chern-Simons terms, we have constructed two finite parameters in the FFBRST transformations. These parameters are only mixed due to the matter terms. The BRST transformations of this theory have been studied in both linear as well as non-linear gauges. After analysing both the linear and non-linear BRST transformations, a finite field dependent version of these transformations has been developed. It has been shown that these two gauges can be related to each other via FFBRST transformations.

Multiple D2-brane action has been derived from a multiple M2-brane action by means of a novel Higgs mechanism [43, 44, 45, 46]. In this mechanism a vacuum expectation value is given to a scalar field

which breaks the gauge group $U(N) \times U(N)$ down to its diagonal subgroup. The theory thus obtained is the Yang-Mills theory coupled to matter fields. It would be interesting to start with a gauge fixed ABJM theory in $\mathcal{N} = 1$ superspace and use the novel Higgs mechanics to obtain the Yang-Mills theory coupled to matter fields. It would also be interesting to study the FFBRST transformations of the ABJM theory and the FFBRST transformations of the theory obtained after using the novel Higgs mechanics. It is expected that the FFBRST transformations for the ABJM theory will reduce to the the FFBRST transformations for the Yang-Mills theory coupled to matter fields.

There is a dual symmetry to the BRST symmetry called the anti-BRST symmetry [47, 48]. The finite field version of anti-BRST (anti-FFBRST) symmetry has also been studied [18, 49]. It would be interesting to study this symmetry for the ABJM theory in $\mathcal{N} = 1$ superspace. Furthermore, the ABJM theory in presence of a boundary has also been analysed [50]. In this theory new boundary degrees of freedom have to be added to make this theory gauge invariant. It would be interesting to analyse the FFBRST and anti-FFBRST symmetry for this ABJM theory in presence of a boundary. These transformations can be used to relate the generating functionals in case of ABJM theory in presence of a boundary.

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